Markets for Efficient Public Good Allocation with Social Distancing

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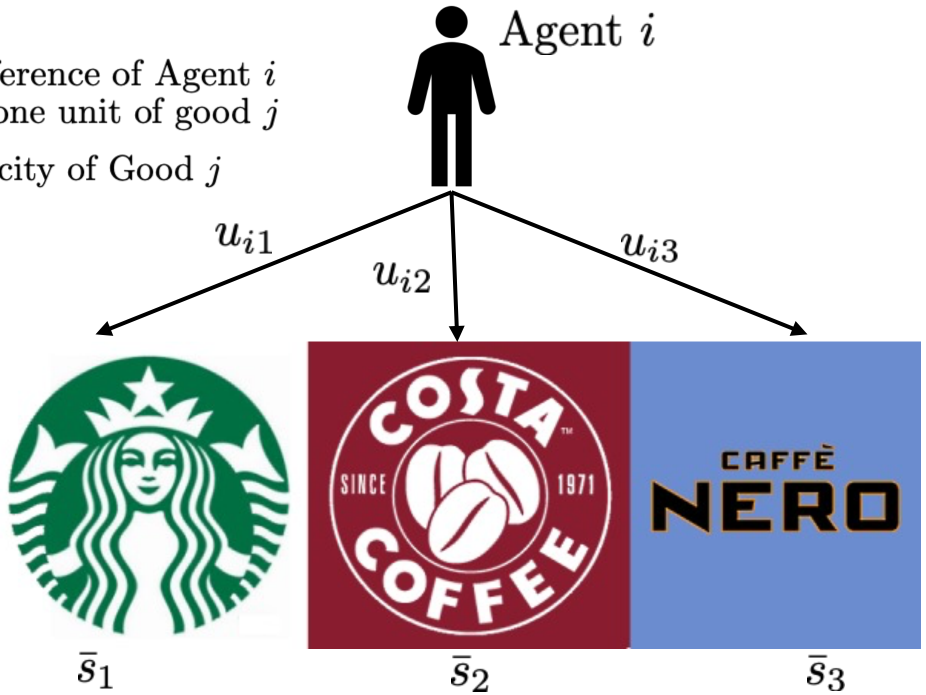
Motivation: Public goods are either over-consumed, or remain unused, as in the Covid-19 pandemic

Our Approach: Design market mechanism using *Fisher Markets* and price "times of use" to achieve an intermediate outcome and in doing so overcome a fundamental limitation of *Fisher Markets* that only consider budget and capacity constraints

Classical Fisher Markets are not expressive enough to capture additional physical constraints

u_{ij} : Preference of Agent i for one unit of good j

$\bar{s}_j$: Capacity of Good j



Social Optimization Problem (SOP):

$$\max_{\mathbf{x}_i, \forall i \in [N]} \sum_i w_i \log \left(\sum_j u_{ij} x_{ij} \right)$$

$$\text{s.t.} \quad \sum_i x_{ij} = \bar{s}_j, \forall j \in [M]$$

$$x_{ij} \geq 0, \forall i, j$$

Capacity Constraint

p_j : Price of Good j = Dual Variable of Constraint j

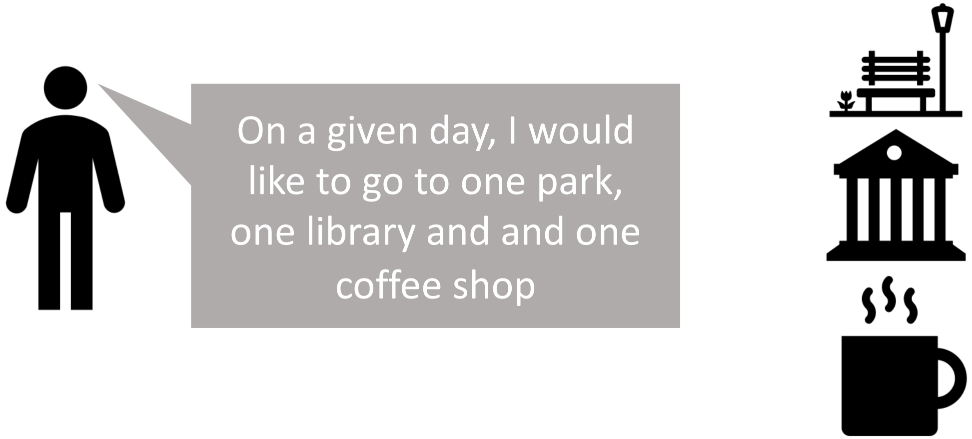
Individual Optimization Problem:

$$\max_{\mathbf{x}_i} \sum_j u_{ij} x_{ij}$$

$$\text{s.t.} \quad \mathbf{p}^T \mathbf{x}_i \leq w_i$$

$$\mathbf{x}_i \geq \mathbf{0}$$

Budget Constraint



With physical constraints, budget perturbations reweight utilities and provide more constrained people a fair chance to use budgets

IOP with Additional Constraints

IOP

$$\max_{\mathbf{x}_i} \sum_j u_{ij} x_{ij}$$
$$\text{s.t. } \mathbf{p}^T \mathbf{x}_i \le w_i$$
$$A_t^{(i)} \mathbf{x}_i \le 1, \forall t \in T_i$$
$$\mathbf{x}_i \ge \mathbf{0}$$

Physical Constraints

Theorem 0: Under mild technical assumptions, the market clearing equilibrium price vector exists.

SOP with Additional Constraints

SOP1

$$\max_{\mathbf{x}_i, \forall i \in [N]} \sum_i w_i \log \left(\sum_j u_{ij} x_{ij} \right)$$
$$\text{s.t. } \sum_i x_{ij} \le \bar{s}_j, \forall j \in [M]$$
$$A_t^{(i)} \mathbf{x}_i \le 1, \forall t \in T_i, \forall i \in [N]$$
$$x_{ij} \ge 0, \forall i, j$$

Theorem 1: The price vector corresponding to the optimal dual variables of the capacity constraint of **SOP1** is not an equilibrium price, i.e., the market clearing first order KKT conditions of **IOP** and **SOP1** are not equivalent.

Budget Perturbed SOP

BP-SOP

Budget Perturbation

$$\max_{\mathbf{x}_i} \sum_i (w_i + \lambda_i) \log \left(\sum_j u_{ij} x_{ij} \right)$$
$$\text{s.t. } \sum_i x_{ij} \le \bar{s}_j, \forall j \in [M]$$
$$A_t^{(i)} \mathbf{x}_i \le 1, \forall t \in T_i, \forall i \in [N]$$
$$x_{ij} \ge 0, \forall i, j$$

r_{it}: Dual Variable of Physical Constraint

Theorem 2: There is a one-to-one correspondence between the equilibrium price vector and a *fixed-point* solution of **BP-SOP**.

Fixed Point of **BP-SOP**: $\lambda_i = \sum_t r_{it}$

We use a fixed-point scheme to determine perturbation constants and establish convergence through experiments

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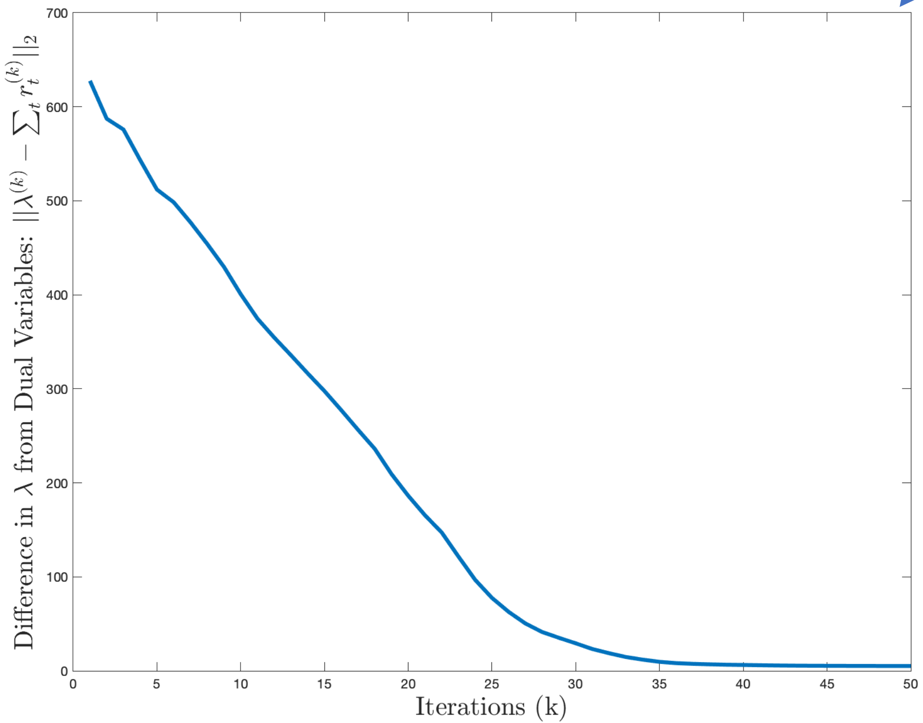
Algorithm 1: Fixed Point Scheme

Input : Tolerance ϵ, Function $G(\cdot)$ to calculate dual variables
Output: Budget Perturbation Parameters λ

$\lambda \leftarrow \mathbf{0}$;
$\mathbf{R} \leftarrow G(\lambda)$;
$q_i \leftarrow \sum_{t=1}^{l_i} r_{it}, \forall i$;
while $\|\lambda - \mathbf{q}\|_2 > \epsilon$ **do**
  $\lambda_i \leftarrow \sum_{t=1}^{l_i} r_{it}, \forall i$;
  $\mathbf{R} \leftarrow G(\lambda)$;
  $q_i \leftarrow \sum_{t=1}^{l_i} r_{it}, \forall i$;
end



Solution Efficiency: For fixed perturbations, **BP-SOP** is a convex optimization problem and so can be solved efficiently

Convergence: Numerical Experiments indicate convergence to a fixed point

Future Research Directions

- Theoretically establish convergence guarantees for fixed-point scheme
- Determine whether fixed point scheme is PPAD-Complete to classify computational complexity of problem
- Extend results to discrete allocations and quantify loss in social objective
- Generalize framework to online setting with sequential customer arrival

arXiv Link: https://arxiv.org/abs/2005.10765