K-Stage Bipartite Matching

- Inspired by applications in matching market

- K-stage bipartite matching (with batch arrival)
  - offline supply nodes
  - K stages, new batch of demand nodes arrives at each stage
  - make irrevocable matching decision

  - goal: maximize the competitive ratio against optimum in-hindsight solution

$$\Gamma(K) \triangleq \sup_{\text{ALG}} \left(\inf_{G, \{w_j\}_{j \in V}} \frac{\mathbb{E}\left[\text{weight}(\text{ALG})\right]}{\text{weight}(\text{OPT-OFFLINE})} \right)$$

SSRN Preprint: 3689448

Batching and Optimal Multi-stage Bipartite Allocations
Yiding Feng (Northwestern), Rad Niazadeh (UChicago)

K-Stage Bipartite Matching

$K = 1$
(fully offline)

$K > 1$
(multi-stage)

$K = |U|$, *batch-size=1*
(fully online)

$\Gamma(1) = 1$

$\Gamma(\infty) = 1 - \frac{1}{e}$

[Formal Research Question]

In the middle-ground regime with $K > 1$ batches, what is the *optimal competitive ratio* achieved by a multi-stage algorithm?

Main Result: $\Gamma(K) = 1 - \left(1 - \frac{1}{K}\right)^K$

Theorem

[Lower-bound] For any $K > 0$, we propose an efficient *(simple)* multi-stage algorithm with competitive ratio no smaller than $\Gamma(K) = 1 - \left(1 - \frac{1}{\mathrm{K}}\right)^K$.

[Upper-bound] For any $K > 0$, there exists an instance for which no multi-stage algorithm has a competitive ratio better than $\Gamma(K) = 1 - \left(1 - \frac{1}{\mathrm{K}}\right)^K$.

We show same results hold when:

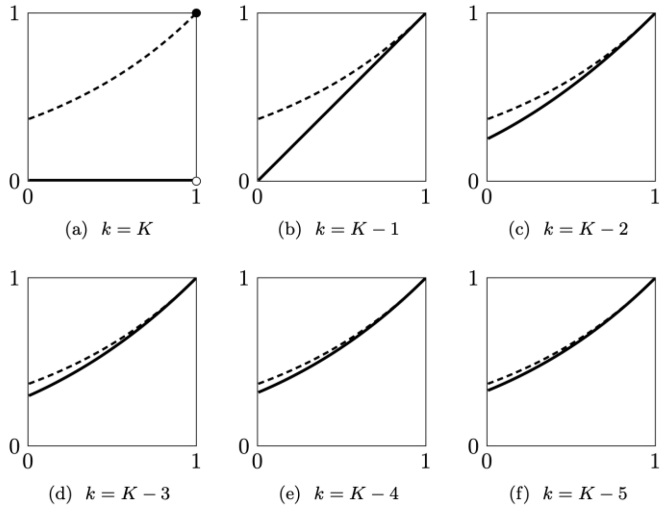
- *Fractional:* Vertex-weighted b-matching, budgeted allocation, assortment
- *Integral:* above settings, with the large budget assumption

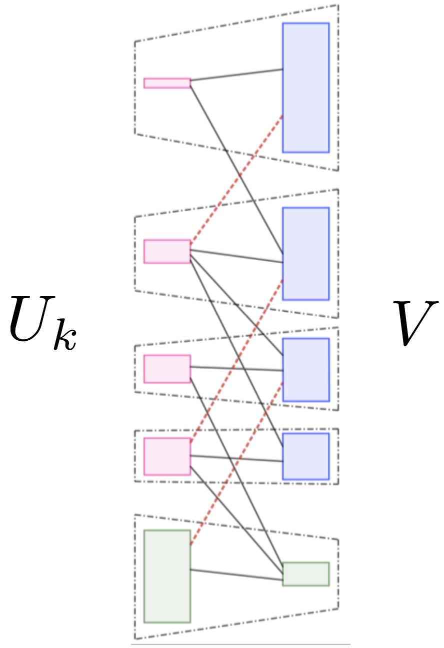
SSRN Preprint: 3689448

Yiding Feng (Northwestern), Rad Niazadeh (UChicago)

Attacking the Problem from a New Perspective

- ***"Polynomial regularized convex-programming matching"***
  - Identifying a particular sequence of polynomials of decreasing degrees, one for each batch
  - Solving resulting regularized max-weight matching convex programs to find a family of balanced allocations
  - Using graph decompositions induced by optimal solutions to come up with a new primal-dual competitive analysis





SSRN Preprint: 3689448