The Smoothed Possibility of Social Choice

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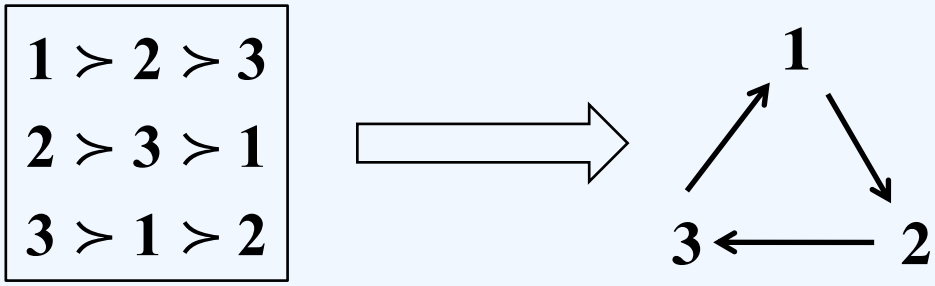
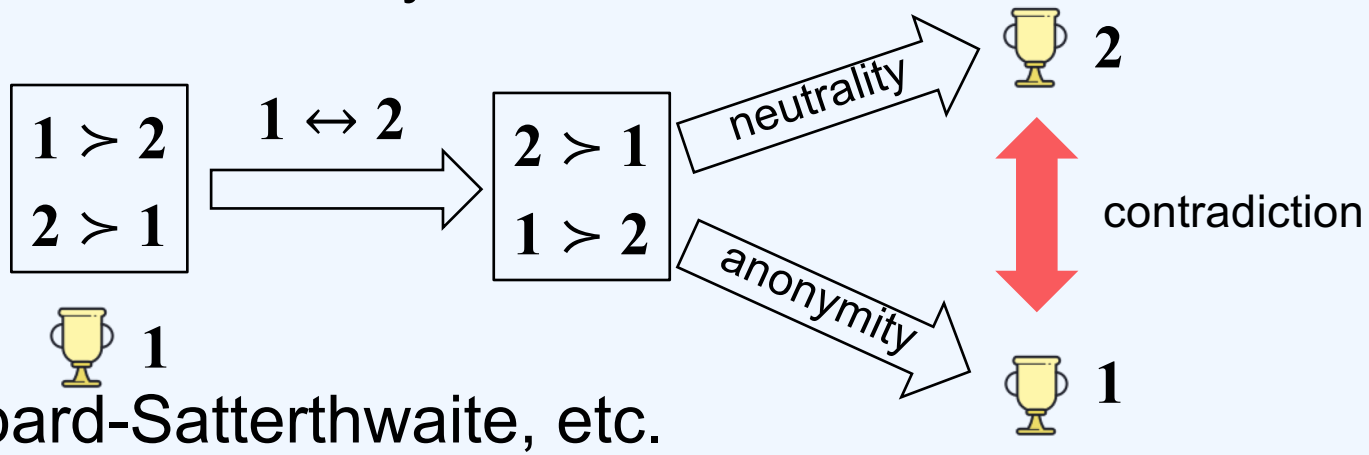
[NeurIPS-2020]

- Impossibilities are mostly worst-case analysis
- Condorcet's Paradox: Pairwise majority is not always transitive.



- ANR Impossibility: No voting rule satisfies Anonymity, Neutrality, and Resolvability.



- Arrow's, Gibbard-Satterthwaite, etc.

Circumventing the impossibilities

	Social Choice	Computer Science
Worst-case	Impossibilities	NP-hardness, O notation $\mathrm{Sup}_P\ \mathrm{Runtime}(P)$
Domain restrictions	Single-peaked preferences ☹ restrictive [Lackner^2 17]	2-SAT ☹ restrictive
Average-case	$\mathrm{Pr}_{P\sim\pi}$ (impossibility) • π : i.i.d., e.g. Impartial Culture ☹ unrealistic [Mulligan & Hunter 03]	Average runtime $\mathrm{E}_{P\sim\pi}\ \mathrm{Runtime}(P)$ ☹ unrealistic
Smoothed analysis	This work*	Spielman & Teng 04 Worst average-case

* A position paper [Baumeister, Hogrebe, Rothe AAMAS-20 blue sky ideas track] independently proposed to study smoothed computational problems in voting, voting paradoxes, and ties, but no technical result was obtained.

Main Results: The Smoothed Possbilities

- Dichotomy theorem: smoothed Condorcet.
  - Smoothed avoidance: either

$$\tilde{S}_{\mathrm{NCC}}(\Pi^n) = 1 - \exp(-\Theta(n))$$ vanishes exponentially fast

  - Smoothed paradox: or

$$\tilde{S}_{\mathrm{NCC}}(\Pi^n) = 1 - \Theta(1)$$ does not vanish

- Dichotomy theorem: smoothed ANR.
  - Smoothed possibility: ANR can be avoided by some rule r

$$\tilde{S}_{\neg\mathrm{ANR}}(r, \Pi^n) = \begin{cases} 1\text{-}O(\mathrm{poly}(n)) & \text{vanishes poly fast} \\ 1\text{-}\exp(\Omega(n)) & \text{vanishes exponentially fast} \end{cases}$$

  - Smoothed impossibility: no resolute rule can do better
- A new easy-to-compute tie-breaking mechanism
  - Most-Popular-Singleton-Ranking (MPSR) tie-breaking
- Main tech: smoothed Pr(PMV in a polyhedron)
  - existing CLT and (anti)-concentration bounds are too coarse