The Smoothed Possibility of Social Choice

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Social Choice

A > B > C
B > A > C
C > A > B

voting rule

Impossibilities as Worst Case Analysis

- Condorcet’s Paradox: Pairwise majority is not always transitive.

1 > 2 > 3
2 > 3 > 1
3 > 1 > 2

- ANR Impossibility: No voting rule satisfies Anonymity, Neutrality, and Resolvability.

1 > 2
2 > 1
1 > 2

- Arrow’s, Gibbard-Satterthwaite, etc.

Too Many Impossibilities

The force and widespread presence of impossibility results generated a consolidated sense of pessimism, and this became a dominant theme in welfare economics and social choice theory in general.


Circumventing the impossibilities

Social Choice  Computer Science

Worst-case Impossibilities  NP-hardness, O notation SupP Runtime(P)

Domain restrictions Single-peaked preferences restrictive [Laecken’2 17] 2-SAT restrictive

Average-case pP,r=r (impossibility) unrealistic [Mulgan & Hunter 03] Average runtime E_{P,r} Runtime(P) unrealistic

Smoothened analysis This paper* [Spielman & Teng 04 Worst average-case]

Smoothed (Complexity) Analysis

[Spielman & Teng 2004]

\[ \theta_{\text{worst}} + \text{noise} \leq \pi \leq \theta_{\text{average}} \]

Sup_{\pi \in \mathbb{P}} E_{P,\pi} \text{Runtime}(P)

Worst-case by Average-case by

Smoothed Social Choice

- Per-profile property X. S_X(r, P) \rightarrow \{0, 1\} s.t.

r satisfies X \leftrightarrow \text{Inf}_{\mathbb{P}} S_X(r, P) = 1

Examples

\[ S_{\text{ANR}}(P)\geq 1, P \text{ does not contain a Condorcet cycle} \]

\[ S_{\text{ANR}}(P, r, P) = 1 \forall \text{ permutation } \eta \text{ over voters } x, \eta((P)) \neq r(P) \forall \text{ permutation } \sigma \text{ over alternatives, } \sigma((P)) = \sigma(r(P)), \text{ and } |r(P)| = 1 \]

Smoothed social choice framework.

- a per-profile property X described by S_X, and

- a set of distributions D over profiles

The smoothed satisfaction of r

\[ \hat{S}_X(r, D) = \text{Inf}_{\mathbb{P}} E_{P,\pi} S_X(r, P) \]

Assumptions in This Paper

- \[ D = \Pi^\kappa \]

\[ \Pi : \text{ a set of distributions over votes for a single agent} \]

- each agent’s “ground truth” preferences

- strictly positive, closed

\[ \theta_{\text{worst}} + \text{noise} \leq \pi \leq \theta_{\text{average}} \]

\[ 1 \leq \pi \leq 2 \]

\[ \text{unrealistic} \]

\[ \text{unrealistic} \]

\[ \text{unrealistic} \]

Smoothed analysis This paper* Spielman & Teng 04 Worst average-case

\[ \text{arbitrarily correlated ground truth, independent noises} \]

* A position paper [Baumeister, Horegbe, Rothe AAMAS-20 blue sky ideas track] independently proposed to study smoothed computational problems in voting, voting paradoxes, and ties, but no technical result was obtained.

Message: Impossibilities can vanish fast

- Question: When?

Our Work: Smoothed Condorcet

- Smoothed avoidance: if CONDITION holds, vanishes exp fast

\[ \hat{S}_{\text{NC}}(\Pi^\kappa) = 1 - \exp(-\Omega(n)) \]

- Smoothed paradox: otherwise, does not vanish

\[ \hat{S}_{\text{NC}}(\Pi^\kappa) = 1 - \Omega(1) \text{ for infinitely many } n \]

Our Work: Smoothed ANR Dichotomy

- Smoothed possibility: ANR can be avoided by some rule r

\[ \hat{S}_{\text{ANR}}(r, \Pi^\kappa) = \begin{cases} 1 - \text{poly}(\Omega(n)) \\ 1 - \text{exp}(\Omega(n)) \end{cases} \]

- Smoothed impossibility: no resolute rule can do better

- A new easy-to-compute tie-breaking mechanism

- Most-Popular-Singleton-Ranking (MPSR) tie-breaking.

- better than lexicographic or fixed-agent tie-breaking w.r.t. S_{\text{ANR}}

Proof Techniques

- Given X

\[ P \text{ satisfies } X \iff \text{Hist}(P) \in \Pi^\kappa \]

- Poisson multinomial variable (PMV)

\[ \text{Hist}(P) \in \Pi^\kappa \]

\[ \text{Pr}(\text{Hist}(P) \in H_1) \]

\[ \text{max}\{ \text{Pr}(\text{Hist}(P) \in H_1), \ldots, \text{Pr}(\text{Hist}(P) \in H_1) \} \leq \text{Pr}(P \text{ satisfies } X) \leq \text{Pr}(\text{Hist}(P) \in H_1) \]

Dichotomy theorem: smoothed Pr(PMV in a polyhedron)

Future Work: Smoothed Social Choice

- Other axioms, properties, impossibilities

- Other problems: judgement aggregation, distortion, matching, resource allocation, etc.