The Smoothed Possibility of Social Choice

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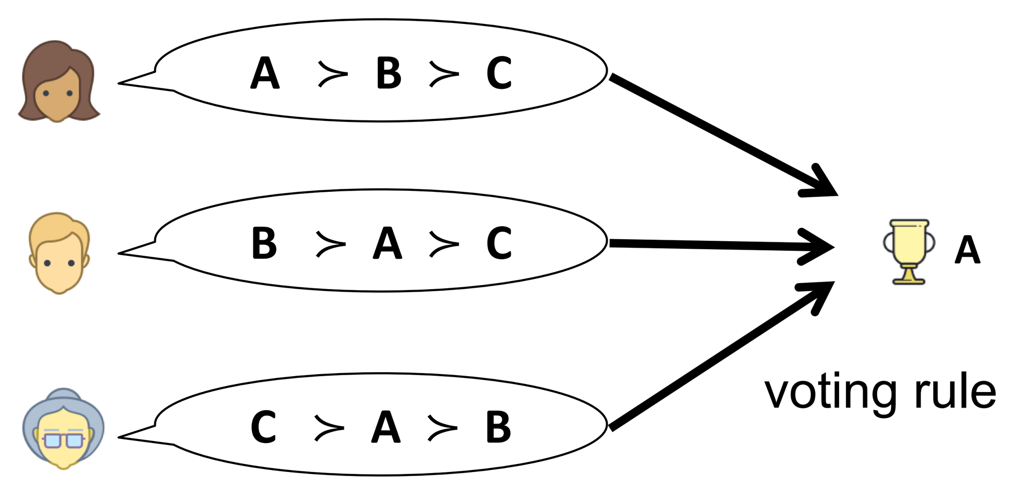
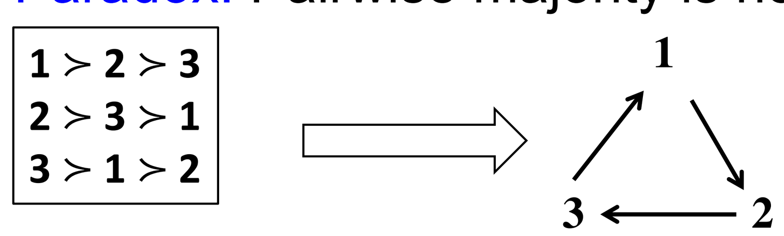
WINE THE 16th CONFERENCE ON WEB AND INTERNET ECONOMICS

Social Choice

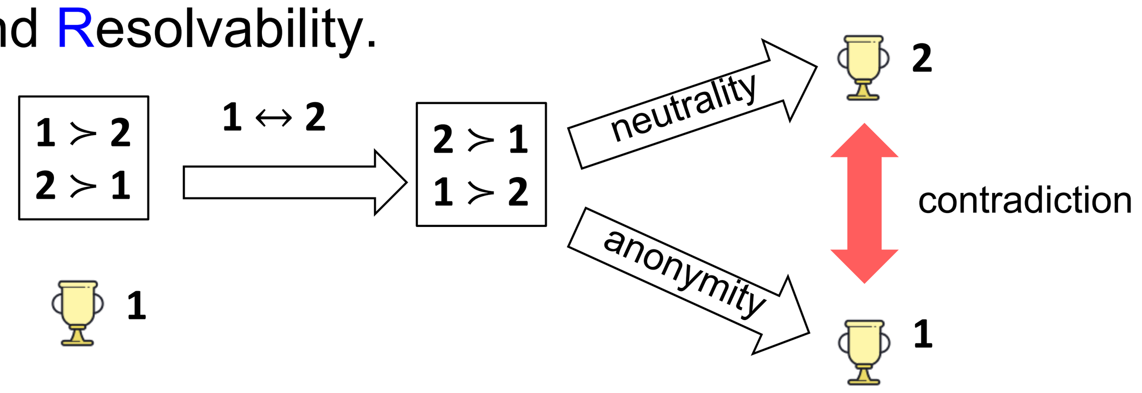


Impossibilities as Worst Case Analysis

- Condorcet's Paradox: Pairwise majority is not always transitive.



- ANR Impossibility: No voting rule satisfies Anonymity, Neutrality, and Resolvability.



- Arrow's, Gibbard-Satterthwaite, etc.

Too Many Impossibilities

The force and widespread presence of impossibility results generated a consolidated sense of pessimism, and this became a dominant theme in welfare economics and social choice theory in general

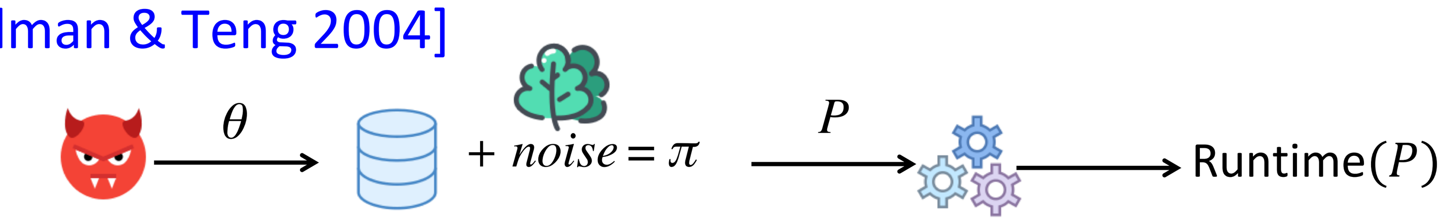
--Amartya Sen. The possibility of social choice. Nobel Prize Lecture & AER. 1999.

Circumventing the impossibilities

	Social Choice	Computer Science
Worst-case	Impossibilities	NP-hardness, O notation $\text{Sup}_P \text{ Runtime}(P)$
Domain restrictions	Single-peaked preferences restrictive [Lackner^2 17]	2-SAT restrictive
Average-case	$\Pr_{P\sim\pi}$(impossibility) • π : i.i.d., e.g. Impartial Culture unrealistic [Mulligan & Hunter 03]	Average runtime $E_{P\sim\pi}\text{ Runtime}(P)$ unrealistic
Smoothed analysis	**This paper***	Spielman & Teng 04 Worst average-case

Smoothed (Complexity) Analysis

[Spielman & Teng 2004]



$\text{Sup}_{\pi\in D}\ E_{P\sim\pi}\ \text{Runtime}(P)$

Worst-case by — Average-case by

"an important step forward in meeting the grand challenge of developing means for predicting the performance of algorithms and heuristics on real data and on real computers.

-----2008 Gödel Prize announcement

Smoothed Social Choice

- Per-profile property X. S_X: (r,profile) →{0,1} s.t.

  r satisfies $X \Leftrightarrow \text{Inf}_P S_X(r,P) = 1$

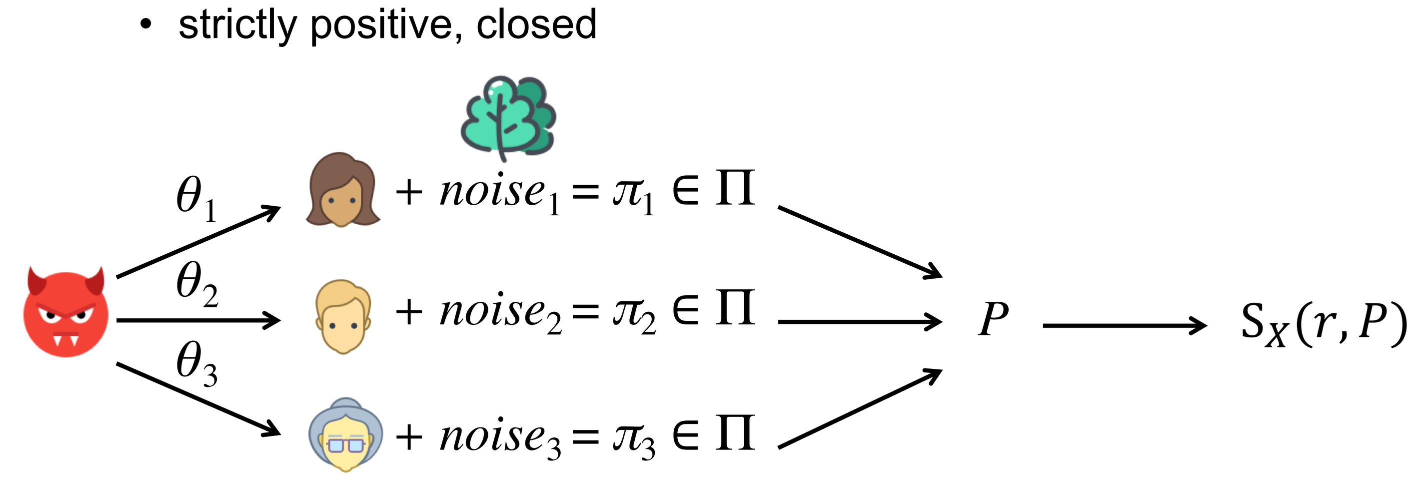
- Examples
  - $S_{NCC}(P)$=1: P does not contains a Condorcet cycle
  - $S_{\neg ANR}(r,P)$=1: ∀ permutation η over voters, $r(\eta(P)) = r(P)$, ∀ permutation σ over alternatives, $r(\sigma(P)) = \sigma(r(P))$, and $|r(P)| = 1$
- Smoothed social choice framework. Given
  - a per-profile property X described by S_X, and
  - a set of distributions D over profiles

The smoothed satisfaction of r

$$\tilde{S}_X(r, D) = \text{Inf}_{\pi\in D}\ E_{P\sim\pi}\ S_X(r,P)$$

Assumptions in This Paper

- $D = \Pi^n$
  - Π : a set of distributions over votes for a single agent
    - each agent's "ground truth" preferences
    - strictly positive, closed



- Arbitrarily correlated ground truth, independent noises

Message: Impossibilities can vanish fast

- Question: When?

Our Work: Smoothed Condorcet

Smoothed avoidance: if CONDITION holds, vanishes exp fast

$$\tilde{S}_{NCC}(\Pi^n) = 1 - \exp(-\Omega(n))$$

Smoothed paradox: otherwise, does not vanish

$$\tilde{S}_{NCC}(\Pi^n) = 1 - \Omega(1) \text{ for infinitely many } n$$

- CONDITION: ∀π ∈ CH(Π), UMG(π) has no weak Condorcet cycle

  convex hull — unweighted majority graph

Our Work: Smoothed ANR Dichotomy

Smoothed possibility: ANR can be avoided by some rule r

$$\tilde{S}_{\neg ANR}(r, \Pi^n) = \begin{cases} 1\text{-}O(\text{poly}(n)) \\ 1\text{-}\exp(\Omega(n)) \end{cases}$$

Smoothed impossibility: no resolute rule can do better

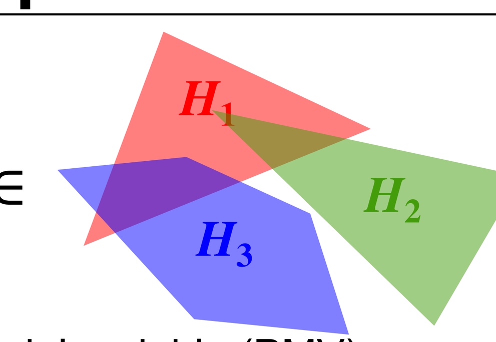
A new easy-to-compute tie-breaking mechanism

- Most-Popular-Singleton-Ranking (MPSR) tie-breaking.
- better than lexicographic or fixed-agent tie-breaking w.r.t. $\tilde{S}_{\neg ANR}$

Proof Techniques

- Given X

  P satisfies $X \Leftrightarrow$ Hist(P) ∈



Poisson multinomial variable (PMV)

- Given $\vec{\pi} \in \Pi^n$,

$\max\{\Pr(\text{Hist}(P) \in H_1), \Pr(\text{Hist}(P) \in H_2), \Pr(\text{Hist}(P) \in H_3)\} \le \Pr_{P\sim\vec{\pi}}(P \text{ satisfies } X) \le \Pr(\text{Hist}(P) \in H_1) + \Pr(\text{Hist}(P) \in H_2) + \Pr(\text{Hist}(P) \in H_3)$

Dichotomy theorem: smoothed Pr(PMV in a polyhedron)

Future Work: Smoothed Social Choice

- Other axioms, properties, impossibilities
- Other problems: judgement aggregation, distortion, matching, resource allocation, etc.

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* A position paper [Baumeister, Hogrebe, Rothe AAMAS-20 blue sky ideas track] independently proposed to study smoothed computational problems in voting, voting paradoxes, and ties, but no technical result was obtained.