On Achieving Fairness and Stability in Many-to-One Matchings

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Shivika Narang[1]| Arpita Biswas[2] | Y Narahari[1]

[1]Indian Institute of Science| [2]Harvard University

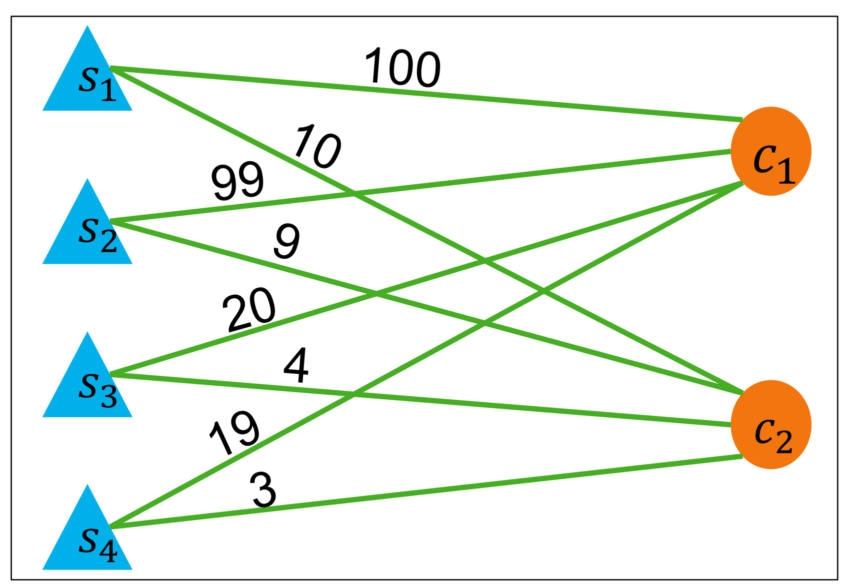
1. Problem

Given: A weighted undirected bipartite graph $G = (S \cup C, E)$ with weights $u(e)$ for all $e \in E$

Goal: Finding **Fair** and **Stable** Many-to-One Matching

2. Introduction

- Many-to-One Matchings with Budgets:



Match n students to m colleges, c_j has budget $b_j \in [n]$.

Isometric utilities $u_{s_i}(c_j) = u_{c_j}(s_i) = u(i,j)$

Global Rankings:

- $c_1 \succ c_2 \succ \cdots \succ c_m$ for all $s_i,\ i \in [n]$
- $s_1 \succ s_2 \succ \cdots \succ s\_n$ for all $c_j, j \in [m]$

3. Definitions

- **Stability**: μ s.t. $\nexists\, s_i, s_{i'}, c_j$ where $u_{s_i}(c_j) > u_{s_i}(\mu(s_i))$ and $u_{c_j}(s_i) > u_{c_j}(s_{i'}), s_{i'} \in \mu(c_j)$
- **EF1:** μ s.t. for all $j, j' \in [m], \exists s_i \in \mu(c_{j'})$ where $u_{c_j}\left(\mu(c_j)\right) \geq u_{c_j}(\mu(c_{j'}) \setminus \{s_i\})$
- **Leximin optimal**: Maximize utility of worst agent, then second worst and so on

4. Contributions

- Envy and Stability are incompatible motivating exploring **leximin**
- Finding leximin optimal stable matching is **NP-Hard**
- $O(mn)$ **time algorithm** finds leximin optimal stable matching over ranked isometric utilities.
- Extends to more general valuations

5. Hurdles to Fairness in Stability

1. EF1 and stability need not coexist. (example 1)
2. Minimizing average envy need not work.

c_1	c_2	E_S	E_C	E_{total}
1-4	-	0	26	26
1-3	4	16	20	36
1,2	3,4	32	12	44
1	2-4	120	38	158
-	1-4	0	238	238

3. Finding leximin optimal stable match is NP-Hard

   Reduction from balanced partition

6. Leximin Optimal Stable Matching for Ranked Isometric Utilities

Algorithm Outline:

1. First match top $n-m+1$ students to c_1 and c_j to s_{n-m+j} for $j \geq 2$ (**student optimal stable matching**)
2. Increase number of students for c_m while feasible and till leximin decrease then c_{m-1} till c_2
   - Checking for decrease is non-trivial: Temporarily demote a student to check if her new utility > college's old one.
   - In case of tie, look ahead is required

$O(mn)$ **time** algorithm

Intuition

Global rankings give structure over stable matchings:

μ is stable $\Leftrightarrow \mu(c_j) = \{s_{h_j+1}, \ldots, s_{h_j+k_j}\}$

where $k_j = |\mu(c_j)|$ and $h_j = \sum_{\{t=1\}}^{\{j-1\}} k_t$ forall $j \in [m]$

i.e. matching preserving ranks.

Ranking+ isometric utilities ⇒students appear in increasing order of rank in leximin tuple of **every** stable matching

∴ Can fix the matchings of students in increasing order of rank.

7. Future Directions

1. Characterizing space of utilities for which tractability holds
2. Exploring other fairness objectives with stability.

References

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* Contact: Shivika Narang, TCS RSP Fellow, IISc (shivika@iisc.ac.in)