A Novel Two-Stage Game Model for the Pricing Management of Edge Computing Resource in Blockchain System

Jinmian Chen, Yukun Cheng* and Zhiqi Xu
School of Business, Suzhou University of Science and Technology,
Suzhou, 215009, China,
645692632@qq.com, ykcheng@amss.ac.cn, joisexzq@163.com

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1 Introduction

Blockchain is a public distributed ledger, which is beneficial to keep the data of transactions integrity, continuity, and consistency. It has generated remarkable public interests through a distributed network without intermediary. The key issue of blockchain is to mine a block, that is to solve a proof-of work (PoW) puzzle, by consuming high computing power and energy. This thus limits the wide adoption of the blockchain in mobile applications [2]. Hence, the edge computing resource paradigm is introduced into the mobile blockchain networks [5], [10], so that the mining tasks of mobile miners can be offloaded to an edge computing resource provider. For the provider, there is an important issue, that is how to price and allocate its computing resource to miners. Jiao et al. [5] applied a combinatorial auction and Luong et al. [6] investigated an optimal auction by deep learning architecture in [1] to make the decision of price and allocation to miners. In addition, Xiong et al. [8] conducted a two-stage Stackelberg game to optimize the profit of the edge computing resource provider and the individual utilities of miners. Inspired by the work in [8], we build a novel two-stage game to model the interaction between the provider and the miners. Similarly, in Stage I, the provider sets the resource price, while in Stage II, we construct an evolutionary game for miners, who pursue the evolutionary stable strategies (ESS) by keeping learning to adjust their low-income strategies and to imitate the others' higher income strategies. Though the backward-induction method, we first explore the different conditions, under which different strategy profiles of miners could be the ESS in the second stage. In addition, for each ESS, the optimal resource demands and payoffs of miners are also derived. In the first stage, we investigate the profit maximization of the edge computing resource provider and then the existence and the uniqueness of the optimal price are validated.

In the following, we first propose the model of block mining assisted by edge computing resource, which has been introduced by [8], [10], and then the two-stage game formulation is presented for the pricing management of edge computing resource.

1.1 Block Mining Assisted by Edge Computing Resource. In this paper, we focus on a public blockchain system, in which the consensus is achieved by PoW-based protocol. All miners must compete to make profit through block mining, i.e., solving PoW puzzle, which shall cost them high computational power. Note that each miner has an initial computational power and some of them may purchase a certain amount of computing resource to speed up the mining process, from an edge computing resource provider. An edge computing resource provider can provide the computing resource through the near-to-end edge devices [8], which is priced by the provider, and the miners who join the market shall decide their demands of computing resource based on the its price. As the assumption in [8], the link between the miners and edge computing units is sufficiently reliable and secured.

To study the pricing management for the edge computing resource, we assume there are two miners, denoted by miner 1 and miner 2, accessing the computing resource. These two miners own their initial computational power with amounts of w_1 and w_2, and their individual computing resource demands from edge computing resource provider are denoted by $q_1(\geq 0)$ and $q_2(\geq 0)$, respectively. Miner i, $i = 1$ or 2, with its computing resource demand q_i and initial computational power w_i, has a relative computing power γ_i [8], with respect to the total computational power of the system, whose formal definition is $\gamma_i(q_1, q_2, w_1, w_2) = \frac{q_i+w_i}{\sum_{i=1}^{2}(q_i+w_i)}$.

Based on the PoW-based consensus protocol, the process of successfully mining a block includes two steps, i.e., the mining step and the propagation step. In the mining step, the probability that miner i competes to be the first one to solve PoW puzzle is directly proportional to its relative computing power γ_i. In the propagation step, the miner must propagate its mined block as soon as possible, otherwise, it is likely to be discarded because of long latency. Here the latency depends on the size of mined block, since the larger size of block brings the longer latency, and leads to a higher chance that the block suffers *orphaned* [3]. Based on the statistics results in [7], the propagation time τ_i of miner i's mined block is linear to its size s_i, i.e., the number of transactions in the block mined by miner i. Thus $\tau_i = zs_i$, where z is a constant, reflecting the impact of s_i on τ_i. In addition, as the arrival rate of a new block is a random variable, following the Poisson distribution with mean $\lambda = \frac{1}{600sec}$ [3], the probability of a block being orphaned is $P_{orphan}(s_i) = 1 - e^{-\lambda \cdot zs_i}$. So the probability of successfully mining a block by miner i and such a block reaches the consensus is $P_i(\gamma_i, s_i) = \gamma_i(1 - P_{orphan}(s_i)) = \gamma_i e^{-\lambda zs_i}$.

To incentive the miners to mine block truthfully, the first one to successfully mines a block that reaches the consensus would earn a reward. This reward consists of a fixed one R (e.g., 6.25 BTC for one block) and the transaction fee, defined as rs_i, which is linear to the block size s_i. Parameter r is a given variable representing the reward factor of a transaction. Since miner i may purchase q_i extra computing resource from the edge computing resource provider, it would cost him $q_i p$, where p is unit resource price settled by the provider. Thus the individual expected utility of miner i, $i = 1$ and 2, is $U_i = (R+rs_i)\gamma_i e^{-\lambda zs_i} - pq_i$.

1.2 A Two-Stage Game Formulation. We model the interaction between the edge computing resource provider and two miners as a novel two-stage game. In this game, the provider is the leader, setting the computing resource price in the first stage. The miners, acting as the followers, decide their evolutionary stable computing resource demands through continuous learning and adjustment in the second stage, based on the price by the leader before. By leveraging the backward induction, we formulate an evolutionary game between two miners in Stage II and an optimization problem for the leader in Stage I.

(1) Miners' stable mining strategies in Stage II. In Stage II, given a unit price p of computing resource, we model the miners' sub-game as an evolutionary game. Each miner has two strategies: one is to buy computing resource, meaning $q_i > 0$; and the other is not to buy, showing $q_i = 0$. Therefore, there are four strategy profiles: (Buy, Buy), (Buy, not Buy), (not Buy, Buy) and (not Buy, not Buy). If miner i chooses the strategy to buy resource, it shall determine the demand q_i to maximize its expected utility, and thus obtain corresponding optimal utility. Different strategy profiles would bring different payoffs (see the payoff matrix Table 1 in Section 3). During the evolutionary game, the miners keep learning to adjust their low-income strategies and to imitate the strategy choice of miner, who has a higher income, until the strategy profile of two miners reaches a stable demand state $(q_1^*(p), q_2^*(p))$.

(2) Provider's optimal pricing strategy in Stage I. The profit of the edge computing resource provider is the difference between the income from charging for miners' computing resource demands and the cost to provide the computing resource. Particularly, the cost is linear to the total supply $(q_1 + q_2)$, defined as $cT(q_1 + q_2)$ by Xiong et al., [8], where T is the time that the miner takes to mine a block; c is the unit cost of electricity. Therefore, the computing resource provider decides the unit price of resource to maximize its profit function as $\Pi(p, q_1, q_2) = p(q_1 + q_2) - cT(q_1 + q_2)$. Hence, the profit maximization problem of the resource provider is formulated as follows:

Provider's sub-game:

$$\max_{p} \Pi(p, q_1, q_2) = p(q_1 + q_2) - cT(q_1 + q_2). \tag{1}$$

2 Equilibrium Analysis

Combining the miners' sub-game and the provider's sub-game, we build a novel two-stage game model for the pricing management of edge computing resource. We conduct the backward induction method to analyze the equilibrium of the two-stage game. That is, we first discuss the evolutionary stable strategies (ESS) of two miners in Stage II, by supposing the resource price is given. After obtaining the ESS of two miners in Stage II, we then come back to explore the optimal pricing of the provider in Stage I.

2.1 Stage II: miners' evolutionary stable strategies. Each miner has two strategies, i.e., buy or not buy, in the evolutionary game. Particularly, if one miner decides to purchase computing resource from the edge computing resource provider under a given price p, then it shall determines the resource demand to maximize his expected utility function. To be specific, if both of two miners choose to buy the computing resource, then miner i, $i = 1$ or 2, has its payoff

$$\pi_{11}^{(i)} = \max_{q_i > 0} U_i = \max_{q_i > 0} \left((R + rs_i) \frac{q_i + w_i}{\sum_{i=1}^{2} (q_i + w_i)} e^{-\lambda z s_i} - pq_i \right);$$

If miner 1 decides to buy resource while miner 2 does not, then

$$\pi_{12}^{(1)} = \max_{q_1>0}\left((R+rs_1)\frac{q_1+w_1}{q_1+w_1+w_2}e^{-\lambda z s_1} - pq_1\right);\quad \pi_{12}^{(2)} = (R+rs_2)\frac{w_2}{q_1+w_1+w_2}e^{-\lambda z s_2}.$$

For the profile (not Buy, Buy), the situation is symmetric and thus

$$\pi_{21}^{(1)} = (R+rs_1)\frac{w_1}{q_2 w_1+w_2}e^{-\lambda z s_1};\quad \pi_{21}^{(2)} = \max_{q_2>0}\left((R+rs_2)\frac{q_2+w_2}{q_2+w_1+w_2}e^{-\lambda z s_2} - pq_2\right);$$

If neither of two miners goes to buy resource, then

$$\pi_{22}^{(i)} = (R+rs_i)\frac{w_i}{w_1+w_2}e^{-\lambda z s_i}, i = 1, 2.$$

The corresponding payoff matrix is shown in the Table 1.

Table 1: Payoff matrix of two miners in the evolutionary model.

miner 1	miner 2: Buy	miner 2: not Buy
Buy	$(\pi_{11}^{(1)}, \pi_{11}^{(2)})$	$(\pi_{12}^{(1)}, \pi_{12}^{(2)})$
not Buy	$(\pi_{21}^{(1)}, \pi_{21}^{(2)})$	$(\pi_{22}^{(1)}, \pi_{22}^{(2)})$

Table 2: The determinants and traces of Jacobian matrix at fixed points.

	$Det\mathbf{J}$	$Tr\mathbf{J}$
(0,0)	$H \cdot K$	$H + K$
(0,1)	$M \cdot (-K)$	$M - K$
(1,0)	$-H \cdot N$	$-H + N$
(1,1)	$M \cdot N$	$-M - N$

Let $A = (R + rs_1)e^{-\lambda z s_1}$ and $B = (R + rs_2)e^{-\lambda z s_2}$. Given the unit computing resource price p. We can obtain the concrete expressions of payoffs by maximizing the utilities of all miners.

For the strategy profile (Buy, Buy), that is both of two miners choose to buy resource, the optimal demands of miner 1 and miner 2 are given as $q_1(p) = \frac{A^2B}{p(A+B)^2} - w_1$ and $q_2(p) = \frac{AB^2}{p(A+B)^2} - w_2$, and the two miners' payoffs are $\pi_{11}^{(1)} = \frac{A^3}{(A+B)^2} + pw_1$ and $\pi_{11}^{(2)} = \frac{B^3}{(A+B)^2} + pw_2$.

For the strategy profile (Buy, not Buy), that is only miner 1 chooses to buy computing resource, while miner 2 does not, then the optimal demand of miner 1 is $q_1(p) = \sqrt{\frac{Aw_2}{p}} - w_1 - w_2$ and the payoffs of two miners are: $\pi_{12}^{(1)} = A - 2\sqrt{Aw_2p} + p(w_1 + w_2)$ and $\pi_{12}^{(2)} = B\sqrt{\frac{w_2p}{A}}$.

For the strategy profile (not Buy, Buy), that is only miner 2 chooses to buy computing resource, while miner 1 does not, then the optimal demand of miner 2 is $q_2(p) = \sqrt{\frac{Bw_1}{p}} - w_1 - w_2$ and the payoffs of two miners are: $\pi_{21}^{(1)} = A\sqrt{\frac{w_1p}{A}}$ and $\pi_{21}^{(2)} = B - 2\sqrt{Bw_1p} + p(w_1 + w_2)$.

For the strategy profile (not Buy, not Buy), meaning $q_1 = q_2 = 0$, then the payoffs of two miners are $\pi_{22}^{(1)} = A\frac{w_1}{w_1+w_2}$ and $\pi_{22}^{(2)} = B\frac{w_2}{w_1+w_2}$.

Equilibrium analysis for evolutionary game. In the established evolutionary game model, each miners has two strategies. Let $x(t)$ and $y(t)$, $0 \le x(t) \le 1$, $0 \le y(t) \le 1$, be the probabilities of miner 1 and miner 2 to play the strategy of buying resource at time t, respectively. Therefore, the possibilities not to buy of miner 1 and miner 2 are $1 - x(t)$ and $1 - y(t)$ at time t, respectively. By using the standard analysis method in [4] and [9] to explore the evolutionary stable strategies (ESS) of an evolutionary game model, we build a replicator dynamics system as

$$\begin{cases} F(x,y) = \frac{dx}{dt} = x(1-x)\left\{\left[(\pi_{11}^{(1)} - \pi_{21}^{(1)}) - (\pi_{12}^{(1)} - \pi_{22}^{(1)})\right] y + (\pi_{12}^{(1)} - \pi_{22}^{(1)})\right\}; \\ G(x,y) = \frac{dy}{dt} = y(1-y)\left\{\left[(\pi_{11}^{(2)} - \pi_{12}^{(2)}) - (\pi_{21}^{(2)} - \pi_{22}^{(2)})\right] x + (\pi_{21}^{(2)} - \pi_{22}^{(2)})\right\}. \end{cases} \tag{2}$$

Based on the stability theorem of the differential equations, all the solutions satisfying $F(x,y) = 0$ and $G(x,y) = 0$ are the fixed equilibrium points of the replicator dynamic system (2). So the fixed equilibrium points of this system are: $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, and (x^*, y^*), where $x^* = \frac{\pi_{22}^{(2)} - \pi_{21}^{(2)}}{(\pi_{11}^{(2)} - \pi_{12}^{(2)}) - (\pi_{21}^{(2)} - \pi_{22}^{(2)})}$; $y^* = \frac{\pi_{22}^{(1)} - \pi_{12}^{(1)}}{(\pi_{11}^{(1)} - \pi_{21}^{(1)}) - (\pi_{12}^{(1)} - \pi_{22}^{(1)})}$, if $x^* \in [0,1]$ and $y^* \in [0,1]$.

Let us construct the Jacobian matrix $\mathbf{J} = \begin{bmatrix} \frac{\partial F(x,y)}{\partial x} & \frac{\partial F(x,y)}{\partial y} \\ \frac{\partial G(x,y)}{\partial x} & \frac{\partial G(x,y)}{\partial y} \end{bmatrix}$ of replicator dynamic system (2). Based on the results in [4], given a point (x,y), if the determinant $DetJ(x,y) > 0$ and the trace $TrJ(x,y) < 0$, then this point is an ESS. First of all, the fixed point (x^*, y^*) cannot be an ESS, due to $TrJ(x^*, y^*) = 0$. Next we shall explore different ESS under different price p. To simplify the discussion, let us denote

$$\begin{aligned} H &= \pi_{12}^{(1)} - \pi_{22}^{(1)} = \tfrac{Aw_2}{w_1+w_2} - 2\sqrt{Aw_2}\sqrt{p} + p(w_1+w_2) = \left(\sqrt{w_1+w_2} - \sqrt{\tfrac{Aw_2}{w_1+w_2}}\right)^2 \geq 0; \\ K &= \pi_{21}^{(2)} - \pi_{22}^{(2)} = \tfrac{Bw_1}{w_1+w_2} - 2\sqrt{Bw_1}\sqrt{p} + p(w_1+w_2) = \left(\sqrt{w_1+w_2} - \sqrt{\tfrac{Bw_1}{w_1+w_2}}\right)^2 \geq 0; \\ M &= \pi_{11}^{(1)} - \pi_{21}^{(1)} = \tfrac{A^3}{(A+B)^2} - A\sqrt{\tfrac{w_1}{B}}\sqrt{p} + pw_1; \\ N &= \pi_{11}^{(2)} - \pi_{12}^{(2)} = \tfrac{B^3}{(A+B)^2} - B\sqrt{\tfrac{w_2}{A}}\sqrt{p} + pw_2. \end{aligned}$$

Then the determinants and traces of Jacobian matrix $\mathbf{J}$ at different fixed points are listed in Table 2.

As stated before, given a point (x,y), if the determinant $DetJ(x,y) > 0$ and the trace $TrJ(x,y) < 0$, then this point is an ESS. Based on the expressions of H, K, M and N, it is not hard to see $(0,0)$ cannot be an ESS. Point $(0,1)$ or $(1,0)$ are the ESS, if and only if $M < 0$ or $N < 0$, respectively. In addition, $(1,1)$ is an ESS, if and only if $M > 0$ and $N > 0$. Thus from the necessary and sufficient conditions for different ESS and the expressions of H, K, M and N, we can explores the conditions under which $(1,1)$, $(0,1)$ and $(1,0)$ are the ESS in Table 3 and 4, respectively.

2.2 Stage I: provider's optimal pricing. Based on the ESS of the evolutionary game in Stage II, the leader of the two-stage game, i.e., the computing resource provider, would optimize pricing strategy to maximize its profit (1).

To be specific, if $(1,1)$ is an ESS under the conditions in Table 3, then the demands of two miners are $q_1(p) = \frac{A^2B}{p(A+B)^2} - w_1$ and $q_2(p) = \frac{AB^2}{p(A+B)^2} - w_2$. So the profit function of provider is $f(p) = (p - cT)(q_1(p) + q_2(p)) = (p - cT)\left[\frac{AB}{p(A+B)} - (w_1 + w_2)\right]$. Thus at $p^* = \sqrt{\frac{ABcT}{(A+B)(w_1+w_2)}}$, $f'(p^*) = 0$ and $f''(p^*) < 0$, implying p^* is the unique optimal price, if it satisfies the conditions in Table 3.

If $(0,1)$ is the ESS under the conditions in Table 4, then the demands are $q_1(p) = 0$ and $q_2(p) = \sqrt{\frac{Bw_1}{p}} - (w_1 + w_2)$ and the profit function is $f(p) =$

Table 3. The conditions under which (1,1) is the ESS

$A > B$	$Aw_2 > Bw_1$		$p > \frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2}, p < \frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2}$, *or* $\frac{AB^2}{w_2} \cdot \frac{1}{(A+B)^2} < p < \frac{A^2B}{w_1} \cdot \frac{1}{(A+B)^2}$;
	$Aw_2 < Bw_1$	$A^3w_2 > B^3w_1$	$p > \frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2}, p < \frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2}$;
		$A^3w_2 < B^3w_1$	$p > \frac{AB^2}{w_2} \cdot \frac{1}{(A+B)^2}, p < \frac{A^2B}{w_1} \cdot \frac{1}{(A+B)^2}$, *or* $\frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2} < p < \frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2}$ (only if $A^4w_2 < B^4w_1$).
$A < B$	$Aw_2 < Bw_1$		$p > \frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2}, p < \frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2}$, *or* $\frac{A^2B}{w_1} \cdot \frac{1}{(A+B)^2} < p < \frac{AB^2}{w_2} \cdot \frac{1}{(A+B)^2}$;
	$Aw_2 > Bw_1$	$A^3w_2 < B^3w_1$	$p > \frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2}, p < \frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2}$;
		$A^3w_2 > B^3w_1$	$p > \frac{A^2B}{w_1} \cdot \frac{1}{(A+B)^2}, p < \frac{AB^2}{w_2} \cdot \frac{1}{(A+B)^2}$, *or* $\frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2} < p < \frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2}$ (only if $A^4w_2 > B^4w_1$).
$A = B$			$p > 0$.

Table 4. The conditions under which (0,1) or (1,0) are the ESS

	$(0,1)$	$(1,0)$
$A > B$	$\frac{A^2B}{w_1} \cdot \frac{1}{(A+B)^2} < p < \frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2}$	$\frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2} < p < \frac{AB^2}{w_2} \cdot \frac{1}{(A+B)^2}$
$A < B$	$\frac{A^4}{Bw_1} \cdot \frac{1}{(A+B)^2} < p < \frac{A^2B}{w_1} \cdot \frac{1}{(A+B)^2}$	$\frac{AB^2}{w_2} \cdot \frac{1}{(A+B)^2} < p < \frac{B^4}{Aw_2} \cdot \frac{1}{(A+B)^2}$
$A = B$	ϕ	ϕ

$(p-cT)\left[\sqrt{\frac{Bw_1}{p}} - (w_1 + w_2)\right]$. It is not hard to see $f'(p) = \frac{1}{2}\sqrt{\frac{Bw_1}{p}} + \frac{cT}{2}\sqrt{\frac{Bw_1}{p^3}} - (w_1 + w_2)$ and $f''(p) < 0$. Due to the concave of $f''(p)$, the provider is able to achieve the maximum profit with the unique optimal price, if it satisfies the conditions in Table 4.

If $(1, 0)$ is the ESS under the conditions in Table 4, then the demands are $q_1(p) = \sqrt{\frac{Aw_2}{p}} - (w_1 + w_2)$ and $q_2(p) = 0$ and the profit function is $f(p) = (p - cT)\left[\sqrt{\frac{Aw_2}{p}} - (w_1 + w_2)\right]$. So $f'(p) = \frac{1}{2}\sqrt{\frac{Aw_2}{p}} + \frac{cT}{2}\sqrt{\frac{Aw_2}{p^3}} - (w_1 + w_2)$ and $f''(p) < 0$. Similarly, the provider is able to achieve the maximum profit with the unique optimal price, if it satisfies the conditions in Table 4.

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